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# Graph Theory Quiz 1 (May 28, 2019)

Open book, open nodes, open neighbor.

You and your friends Mehmet, Kamesh, Siva, and Amelia compete with each other in a Brazilian Jiu Jitsu competition, with the following outcomes:

You beat Siva and Kamesh

Amelia beats Siva and Mehmet

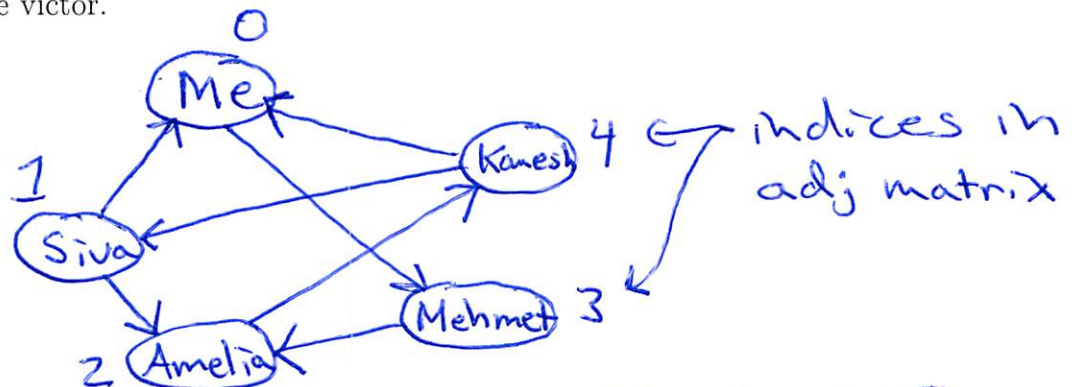
Mehmet beats you

Kamesh beats Amelia

Siva beats Kamesh

Because you and Amelia have the same number of victories, you both agree that obviously the overall winner should be determined as the one with the highest PageRank calculated on a graph modeling the competition.

- First, create this graphical model where each competitor is a vertex and an edge exists between any two competitors that competed. *Orient* the graph so each edge points to the victor.



- Next, compute the transition probability matrix  $M$ .

$$M = (D^{-1}A)^T$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \end{bmatrix}$$

$D^{-1}$

$A$

1

$$M = \begin{bmatrix} 0 & 1/2 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/2 \\ 0 & 1/2 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

3. Finally, initialize PageRanks equally among competitors and calculate a single iteration using the linear algebraic formulation. Who wins?

$$\bar{p}_0 = \begin{bmatrix} 1/5 \\ 1/5 \\ 1/5 \\ 1/5 \\ 1/5 \end{bmatrix}$$

$$\bar{p}_1 = M \bar{p}_0$$

$$\bar{p}_1 = \begin{bmatrix} 0 & 1/2 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 & 1/2 \\ 0 & 1/2 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/5 \\ 1/5 \\ 1/5 \\ 1/5 \\ 1/5 \end{bmatrix} = \begin{bmatrix} 2/10 \\ 1/10 \\ 3/10 \\ 2/10 \\ 2/10 \end{bmatrix}$$

highest PageRank  
is index 2 = Amelia

Amelia wins